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B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2016.

Fourth Semester

Electronics and Communication Engineering

MA 2261/MA 45/MA 1253/080380009/10177 PR 401 — PROBABILITY AND RANDOM PROCESSES

(Common to Biomedical Engineering)

(Regulations 2008/2010)

Time : Three hours

Maximum : 100 marks

(Use of Statistical tables is permitted)

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If the random variable X takes the values 1, 2, 3 and 4 such that $2P(x=1)=3P(x=2)=P(x=3)=5P(x=4)$ find the probability distribution.

2. A die is tossed until 6 appear. What is the probability that it must be tossed more than 5 times?

3. If X and Y are independent RVs then show that $E(Y/X)=E(Y)$ and $E(X/Y)=E(X)$.

4. If $X_1, X_2, ...X_n$ are Poisson variates with parameter $\lambda = 2$, use the CLT to estimate $P(120 \leq S_n \leq 160)$ where $S_n = X_1 + X_2 + ...X_n =$ and $n = 75$.

5. In the fair coin experiment we define the process $\{X(t)\}$ as follows.

$$X(t) = \begin{cases} \sin \pi t & \text{if head shows} \\ 2t & \text{if tail shows.} \end{cases}$$

Find $E(x(t))$ and find $f(x,t)$ for $t = 0.25$

6. Patients arrive randomly and independently at a doctor's consulting room from 8 A.M. at an average rate of 1 every 5 minutes. The waiting room can hold 12 persons. What is probability that the room will be full when the doctor arrives at 9 A.M?

7. Define Wiener Khintchine relation and state any two properties of cross spectral density.

8. An auto correlation function $R(\tau)$ of $\{x(t): \tau \in T\}$ is given by $C.e^{-\alpha|\tau|}$; $C > 0$; $\alpha > 0$ obtain the spectral density of $X(t)$.

9. Define linear time invariant system.

10. If the power spectral density of a WSS process is given by $S_{XX}(w) = \begin{cases} 1 + w^2, & |w| < 1 \\ 0, & |w| > 1 \end{cases}$ find the auto correlation function of the process.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Define Binomial distribution. A coin is tossed until the first head occurs. Assuming that the tosses are independent and the probability of a head occurring is p. find the value of p so that the probability that an odd number of tosses are required is equal to 0.6. Can you find a value of p so that the probability is 0.5 that an odd number of tosses are required? (8)

(ii) Define normal distribution. The time in hours required to repair a machine is exponentially distributed with parameter $\lambda = 1/2$. What is the probability that the repair time exceeds $2\,h$, what is the conditional probability that a repair takes at least $10\,h$ given that its duration exceeds $9\,h$? (8)

Or

(b) (i) Derive the M.G.F of a Poisson random variable. Also find mean and variance of it. State and Prove additive property of Poisson distribution. (8)

(ii) Define Uniform distribution. Consider a random variable X with density function $f_x(x) = e^{-3|x|}$, $-\infty < x < \infty$. Let $Y = e^X$. Find the p.d.f. for Y. (8)

12. (a) (i) If the joint pdf of (X, Y) is given by $f(x, y) = 2$, $0 \le x \le y \le 1$. Find the marginal density functions of X and Y, Conditional densities of $f(x/y)$ and $f(y/x)$ and conditional variance of X given $Y = \frac{1}{2}$. (8)

(ii) For the following bivariate distribution calculate the value of correlation coefficient.

Y/X	0	1	2	3
1	5/48	7/48	–	–
2	9/48	5/48	5/48	–
3	1/12	1/12	1/12	5/48

Or

(b) (i) The joint p.d.f of a two dimensional random variable (X,Y) is $f(x,y)=\begin{cases} e^{-y}, & 0<x<y<\infty \\ 0, & \text{otherwise.} \end{cases}$ Find the regression curves of means. (8)

(ii) The random variable (X,Y) has the joint p.d.f $f(x,y)=\begin{cases} 24xy, & x\geq 0, y\geq 0, x+y\leq 1 \\ 0, & \text{otherwise} \end{cases}$ show that $U=X+Y$, $V=X/Y$ are independent. (8)

13. (a) (i) Given a RV Ω with density $f(w)$ and another RV ϕ, uniformly distributed in $(-\pi,\pi)$ and independent of Ω and $x(t)=a\cos(\Omega t+\phi)$ prove that $\{x(t), t>0\}$ is a WSS process. (8)

(ii) Suppose $x(t)$ is a normal process with mean $\mu(t)=3$ and $c(t_1,t_2)=4e^{-0.2|t_1-t_2|}$ find the probability that $x(5)\leq 2$ and $|x(8)-x(5)|\leq 1$. (8)

Or

(b) Define semi random telegraph signal process and random telegraph signal process and prove also that the former is evolutionary and the latter is wide sense stationary.

14. (a) (i) Given that a process $x(t)$ has an auto correlation function $R_{xx}(\tau)=Ae^{-\alpha|\tau|}\cos(w_0\tau)$ where $A>0$, $\alpha>0$ and w_0 are real constants, find the power spectral density of $x(t)$. (8)

(ii) The cross power spectrum of real random processes $x(t)$ and $y(t)$ is given by $S_{xy}(w)=\begin{cases} a+j.bw; & |w|<1 \\ 0 & \text{elsewhere} \end{cases}$ find the cross correlation function. (8)

Or

(b) (i) $\{X(t)\}$ is a stationary random process with power spectral density $S_{xx}(w)$ and $Y(t)$ is another independent random process $Y(t)=A\cos(w_0t+\theta)$ where θ is a random variable uniformly distributed over $(-\pi,\pi)$. Find the P.S.D of $\{Z(t)\}$ where $Z(t)=X(t)Y(t)$. (8)

(ii) If $X(t)$ and $Y(t)$ are uncorrelated random processes then find the power spectral density of Z if $Z(t)=X(t)+Y(t)$. Also find the cross spectral density $S_{xz}(w)$ and $S_{yz}(w)$. (8)

15. (a) (i) A random process $X(t)$ having the auto correlation function $R_{xx}(\tau) = pe^{-\alpha|\tau|}$, where p and α are real positive constants, is applied to the input of the system with impulse response $H(t) = \begin{cases} e^{-\lambda t}, & t > 0 \\ 0, & t < 0 \end{cases}$ where λ is a real positive constant. Find the auto correlation function of the network response $Y(t)$. (8)

(ii) Consider a White Gaussian noise of zero mean and power spectral density $N_0/2$ applied to a low pass RC filter whose transfer function $H(f) = 1/(1 + i2\pi fRC)$. Find the auto correlation function of the output random process. Also find the mean square value of the output process. (8)

Or

(b) (i) If the input of a time invariant stable linear system is a WSS process then the output will also be a WSS process. (8)

(ii) Find the power spectral density of Binary Transmission process where auto correlation function is $R(\tau) = \begin{cases} 1 - \frac{|\tau|}{T}; & |\tau| \le T \\ 0 & \text{otherwise.} \end{cases}$ (8)